

Reg. No. :

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

Question Paper Code : 71735

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2017.

Fifth /Sixth Semester

Electronics and Communication Engineering

EC 6502 — PRINCIPLES OF DIGITAL SIGNAL PROCESSING

(Common to Biomedical Engineering, Medical Electronics)

(Regulations 2013)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. What is the relation between DTFT and DFT?
2. Compute the DFT of the sequence $x(n) = \{1, -1, 1, -1\}$.
3. What are the requirements for the digital filter to be stable and casual?
4. Discuss the need for prewarping.
5. What is Gibbs phenomenon?
6. Compare Hamming window with Blackmann window.
7. What are the methods used to prevent overflow?
8. What is meant by "dead band" of the filter?
9. Define adaptive filtering.
10. List the applications of multirate signal processing.

PART B — (5 × 16 = 80 marks)

11. (a) Compute the DFT for the sequence $\{1, 2, 3, 4, 4, 3, 2, 1\}$. Using radix - 2 DIF - FFT algorithm. (16)

Or

- (b) In an LTI system the input $x(n) = \{1, 1, 2, 1\}$ and the impulse response $h(n) = \{1, 2, 3, 4\}$. Perform the circular convolution using DFT and IDFT. (16)

12. (a) Design a digital Butterworth filter with the following specifications

$$0.707 \leq |H(e^{j\omega})| \leq 1, 0 \leq \omega \leq 0.5\pi$$

$$|H(e^{j\omega})| \leq 0.2, 0.75\pi \leq \omega \leq \pi$$

Determine system function $H(z)$ for a Butterworth filter using Bilinear transformation. (16)

Or

- (b) Determine the system function of the lowest order digital Chebyshev filter with the following specifications, 3db ripple in the pass band $0 \leq \omega \leq 0.2\pi$ and 25db attenuation in the stop band $0.45\pi \leq \omega \leq \pi$. (16)

13. (a) Design a HPF with the following frequency response

$$H_d(e^{j\omega}) = 1 \text{ for } \pi/4 \leq |\omega| \leq \pi$$

$$= 0 \text{ for } |\omega| \leq \pi/4$$

of length $N = 11$ using Hanning window. (16)

Or

- (b) Determine the coefficients of a linear phase FIR filter of length $N = 15$ which has a symmetric unit sample response and a frequency response that satisfies the conditions. (16)

$$H(2\pi k/15) = 1; \text{ for } k = 0, 1, 2, 3$$

$$= 0; \text{ for } k = 4, 5, 6, 7$$

14. (a) Two first order filters are connected in cascaded whose system functions of the individual sections are $H_1(z) = 1/(1 - 0.5z^{-1})$ and $H_2(z) = 1/(1 - 0.6z^{-1})$. Determine the overall output noise power. (16)

Or

- (b) Explain the characteristics of limit cycle oscillations with respect to the system described by the difference equation $y(n) = 0.95y(n-1) + x(n)$. Determine the dead band. (16)
15. (a) Explain the concept of decimation by a factor D and interpolation by factor I. With help of equation explain sampling rate conversion by a rational factor I/D. (16)

Or

- (b) Explain the operation of adaptive filter with suitable diagrams and equations. (16)

April - May 2017

1. What is the relation between DTFT and DFT?

$$X(e^{j\omega}) = X(k) \rightarrow X(e^{j\omega_k}) \rightarrow X(k)$$

$$\omega_k = \frac{2\pi k}{N}$$

2. Compute the DFT of the sequence $x(n) = \{1, -1, 1, -1\}$

$$X(K) = \{0, 0, 4, 0\}$$

3. What are the requirements for the digital filter to be stable and causal?

1. The digital transfer function $H(Z)$ should be rational function of Z and the coefficient of Z should be real
2. The poles should lie inside the unit circle in the Z plane
3. The number of zeros should be less than or equal to number of poles

4. Discuss the need for prewarping.

In IIR filter design using bilinear transformation, the conversion of the specified digital frequencies to analog frequencies is called prewarping.

The prewarping is necessary to eliminate the effect of warping on the amplitude response.

5. What is Gibb's Phenomenon ?

In FIR filter design by Fourier series method or Rectangular window method the infinite duration impulse response is truncated to finite duration impulse response. The abrupt truncation of impulse response introduce oscillations in the pass band and stop band. The effect is known as Gibbs oscillation.

6. Compare Hamming window with Blackman Window

Hamming window	Blackman Window
The width of mainlobe in window spectrum is $8\pi/N$	The width of mainlobe in window spectrum is $12\pi/N$
The maximum sidelobe magnitude in window spectrum is -41 dB	The maximum sidelobe magnitude in window spectrum is -58 dB
The higher value of side - lobe attenuation is achieved at the expense of constant attenuation at higher frequencies	The higher value of side - lobe attenuation is achieved at the expense of increased main lobe width



7. What are the methods to prevent overflow?

- i) Truncation
- ii) Rounding
- iii) Scaling

8. What is meant by dead band of the filter ?

In a limit cycle, the amplitudes of the output are confined a range of value and this range of value is called dead band of the filter.

9. Define adaptive filtering

The adaptive filter has the property of self-optimization. It is a recursive time varying filter characterized by a set of adjustable coefficients.

10. List the applications of multi – rate signal processing

- 1. Sub band coding of speech signals and image compression
- 2. Oversampling A/D and D/A converters for high quality digital audio systems and digital storage systems.

ii) a)

PART B

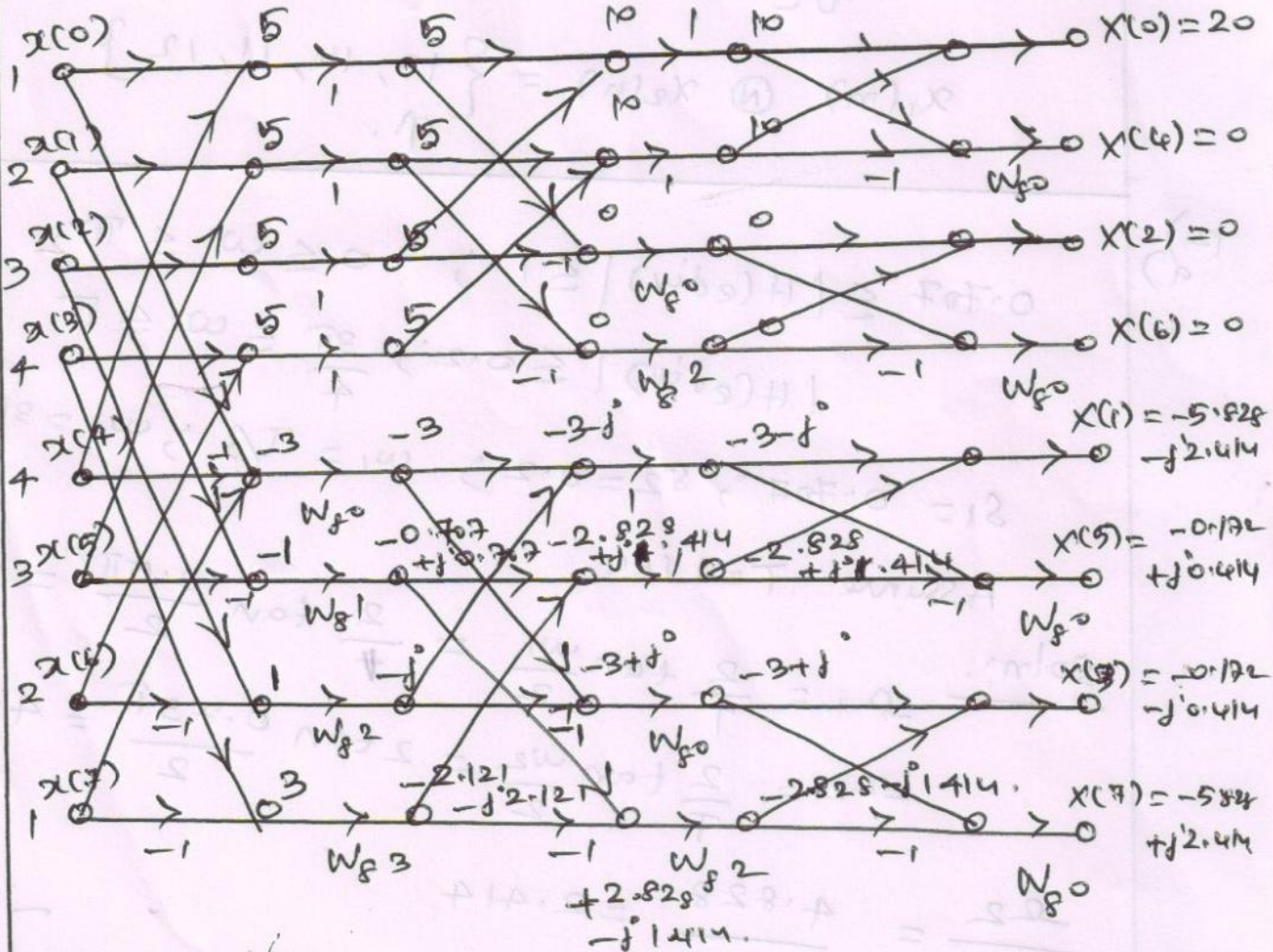
(B)

$$x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$$

$$W_8^0 = 1; \quad W_8^1 = 0.707 - j0.707;$$

$$W_8^2 = -j; \quad W_8^3 = -0.707 - j0.707$$

DIF-FFT



$$X(k) = \{20, -5.828 - j2.414, 0, -0.172 - j0.414, -0.172 + j0.414, 0, -5.828 + j2.414, 0\}$$

ii) b)

Circular Convolution through DFT-IDFT approach:

Steps:

- i) find a point N DFT of $x_1(n)$ and $x_2(n)$ such as $X_1(k)$ and $X_2(k)$
- ii) Multiply $X_1(k)$ and $X_2(k)$

$$X_1(k) = \{5, -1, 1, -1\}$$

$$X_2(k) = \{10, -2+2j, -2, -2-2j\}$$

$$X_1(k) \cdot X_2(k) = \{50, 2-2j, -2, 2+2j\}$$

$$\text{IDFT}_8 [X_1(k) \cdot X_2(k)] = \{13, 14, 11, 12\}$$

$$x_1(m) \otimes x_2(n) = \{13, 14, 11, 12\}$$

12)
a)

$$0.707 \leq |H(e^{j\omega})| \leq 1; \quad 0 \leq \omega \leq \pi/2$$

$$|H(e^{j\omega})| \leq 0.2; \quad \frac{3\pi}{4} \leq \omega \leq \pi$$

$$S_1 = 0.707; \quad S_2 = 0.2; \quad \omega_1 = \pi/2; \quad \omega_2 = 3\pi/4$$

Assume $T = 1$ sec.

Soln:

$$\Omega_1 = \frac{2}{T} \tan \frac{\omega_1}{2} = \frac{2}{1} \tan \frac{0.5\pi}{2} = 2$$

$$\Omega_2 = \frac{2}{T} \tan \frac{\omega_2}{2} = 2 \tan \frac{0.75\pi}{2} = 4.828$$

$$\frac{\Omega_2}{\Omega_1} = \frac{4.828}{2} = 2.414$$

$$N = \frac{1}{2} \left[\frac{\log\left(\frac{1}{S_2^2} - 1\right)}{\log\left(\frac{\Omega_2}{\Omega_1}\right)} \right]$$

$$= \frac{1}{2} \left[\frac{\log\left(\frac{24}{1}\right)}{\log(2.414)} \right]$$

$$N = 1.80 = 2$$

$$\Omega_c = \frac{\Omega_1}{\left[\frac{1}{s_1^2} - 1 \right]^{\frac{1}{2N}}} = \frac{2}{\left[\frac{1}{0.902^2} - 1 \right]^{\frac{1}{4}}} = 2 \text{ rad/sec. } \textcircled{5}$$

$$N = \text{even} \quad \frac{N/2}{k=1} \quad \frac{B_k \Omega_c^2}{s^2 + b_k \Omega_c s + c_k \Omega_c^2}$$

$$H_a(s) = \frac{B_1 \Omega_c^2}{s^2 + b_1 \Omega_c s + c_1 \Omega_c^2}$$

$$B_1 = c_1 = 1; \quad b_1 = 2 \sin\left(\frac{2(1-1)\pi}{2N}\right) \pi$$

$$b_1 = 2 \sin\left(\frac{\pi}{4}\right) = 1.414$$

$$H_a(s) = \frac{1 \times 2^2}{s^2 + 1.414 \times 2 s + 1 \times 2^2}$$

$$H_a(s) = \frac{4}{s^2 + 2.828s + 4}$$

$$\text{Bilinear Transformation } s = \frac{2}{T} \left[\frac{z-1}{z+1} \right] = 2 \left[\frac{z-1}{z+1} \right]$$

$$H(z) = \frac{4}{4 \left[\frac{z-1}{z+1} \right]^2 + 2.828 \left[\frac{z-1}{z+1} \right] + 4}$$

$$H(z) = \frac{4 [z+1]^2}{4 [z-1]^2 + 2.828 [z^2-1] + 4 [z+1]^2}$$

12)
b)

passband ripple = 3db
stop band attenuation = 25db

Chebyshev Filter
design

Taking Anti log on both sides

(6)

$$\delta_1 = 0.707$$

Similarly

$$-20 \log \delta_2 = 25$$

$$\log \delta_2 = -1.25$$

Taking Anti log on both sides we will get

$$\delta_2 = 0.05$$

The desired frequency response specifications are

$$0.707 \leq |H(e^{j\omega})| \leq 1 \quad 0 \leq \omega \leq 0.2\pi$$

$$|H(e^{j\omega})| \leq 0.05 \quad 0.45\pi \leq \omega \leq \pi$$

Assume $T=1$; Take bilinear transformation.

$$\Omega_1 = \frac{2}{T} \tan \frac{\omega_1}{2} = 2 \tan \frac{0.2\pi}{2} = 0.649$$

$$\Omega_2 = \frac{2}{T} \tan \frac{\omega_2}{2} = 2 \tan \frac{0.45\pi}{2} = 1.708$$

$$\Omega_c = \Omega_1 = 0.649$$

$$\frac{\Omega_2}{\Omega_1} = \frac{1.708}{0.649} = 2.63$$

$$\epsilon = \left[\frac{1}{\delta_1^2} - 1 \right]^{0.5} = \left[\frac{1}{0.707^2} - 1 \right]^{0.5} = 1$$

$$N \geq \cosh^{-1} \left\{ \frac{1}{\epsilon} \left[\frac{1}{\delta_2^2} - 1 \right]^{0.5} \right\}$$

$$\cosh^{-1} \left(\frac{\Omega_2}{\Omega_1} \right)$$

$$N \geq \cosh^{-1} \left\{ \frac{1.708}{0.649} \right\} = 3.687 = 2.27 = 3$$

$$y_N = \frac{1}{2} \left\{ \left[\left[\frac{1}{\epsilon^2} + 1 \right]^{\frac{1}{2}} + \frac{1}{\epsilon} \right]^{\frac{1}{2}} - \left[\left[\frac{1}{\epsilon^2} + 1 \right]^{\frac{1}{2}} - \frac{1}{\epsilon} \right]^{\frac{1}{2}} \right\}$$

$$y_3 = \frac{1}{2} \left\{ (2.414)^{\frac{1}{3}} - (2.414)^{-\frac{1}{3}} \right\}$$

$$y_3 = \frac{1}{2} \left\{ 1.3414 - 0.745 \right\}$$

Pg 7

$$y_3 = 0.29$$

$$C_0 = y_3 = 0.29$$

$$C_1 = y_3^2 + \cos^2 \left(\frac{(2K-1)\pi}{2N} \right) = y_3^2 + \cos^2 \frac{\pi}{6}$$

$$C_1 = (0.29)^2 + 0.75 = 0.834$$

$$B_1 = 2 y_3 \sin \left(\frac{\pi}{6} \right) = 2 \times 0.29 \times 0.5 = 0.29$$

$$\frac{1}{(1+\epsilon^2)^{\frac{1}{2}}} = \frac{1}{\sqrt{1+\epsilon^2}}$$

$$1 = \sum_{k=0}^{N-1} \frac{B_k}{C_k}$$

$$1 = \sum_{k=0}^1 \frac{B_k}{C_k}$$

$$1 = \frac{B_0 \cdot B_1}{C_0 \cdot C_1}$$

$$C_0 \cdot C_1 = B_0 \cdot B_1$$

$$0.29 \times 0.834 = B_0 \cdot B_1$$

$$B_0 \cdot B_1 = 0.241$$

$$B_0 = B_1 = \sqrt{0.241} = 0.491 \approx 0.5$$

$N=3$, odd order

$$H_a(s) = \left(\frac{B_0 \omega_c}{s + C_0 \omega_c} \right) \left[\frac{B_1 \omega_c^2}{s^2 + b_1 \omega_c s + C_1 \omega_c^2} \right]$$

$b_1 = 0.5 \times 0.649^2$

$$= \left[\frac{0.3245}{s + 0.1892} \right] \left[\frac{0.21}{s^2 + 0.122s + 0.351} \right]$$

(8)

bilinear transformation

$$s = \frac{2}{T} \left(\frac{z-1}{z+1} \right) = 2 \left[\frac{z-1}{z+1} \right]$$

$$H(z) = \left[\frac{0.3245}{2 \left(\frac{z-1}{z+1} \right) + 0.1892} \right] \left[\frac{0.21}{4 \left[\frac{z-1}{z+1} \right]^2 + 2 \times 0.122 \left[\frac{z-1}{z+1} \right] + 0.351} \right]$$

$$H(z) = \left[\frac{0.3245 [z+1]}{2(z-1) + 0.1892} \right] \left[\frac{0.21 [z+1]^2}{4[z-1]^2 + 0.244(z-1) + 0.351(z+1)} \right]$$

13)
a)

FIR - HPF DEM/m.

$$H_d(e^{j\omega}) = \begin{cases} 1 & \frac{\pi}{4} \leq |\omega| \leq \pi \\ 0 & |\omega| \leq \pi/4 \end{cases}$$

$N=11$; Hanning window:

Soln:

Modified desired frequency response is

$$\left\{ \begin{aligned} H_d(e^{j\omega}) &= 1 & -\pi \leq \omega \leq -\pi/4 \\ &= 1 & \pi/4 \leq \omega \leq \pi \\ &= 0 & |\omega| \leq \pi/4 \end{aligned} \right.$$

$$H_d(m) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) \cdot e^{j\omega m} \cdot d\omega$$

$$= \frac{1}{2\pi} \int_{-\pi}^{-\pi/4} 1 \cdot e^{j\omega m} \cdot d\omega + \frac{1}{2\pi} \int_{\pi/4}^{\pi} 1 \cdot e^{j\omega m} \cdot d\omega$$

$$= \frac{1}{2\pi} \left[\left(\frac{e^{j\omega n}}{j^n} \right)_{-\pi}^{-\pi/4} + \left(\frac{e^{j\omega n}}{j^n} \right)_{\pi/4}^{\pi} \right] \quad (9)$$

$$= \frac{1}{2\pi j^n} \left[e^{-j\frac{n\pi}{4}} - e^{-j\pi} + e^{j\pi} - e^{j\frac{n\pi}{4}} \right]$$

$$= \frac{1}{n\pi} \left[\frac{e^{j\frac{n\pi}{4}} - e^{-j\frac{n\pi}{4}}}{2j} - \frac{e^{j\pi} - e^{-j\pi}}{2j} \right]$$

$$= \frac{1}{n\pi} \left[\sin \frac{n\pi}{4} - \sin \frac{n\pi}{4} \right] \quad n \neq 0$$

$$h(n) = h_d(n) \times W_{Hann}(n)$$

If the filter is symmetrical; ~~about~~ about origin; then the window is

$$W_{Hann}(n) = \begin{cases} 0.5 + 0.5 \cos \frac{2\pi n}{N-1} \left(\frac{N-1}{2} \leq n \leq \frac{N-1}{2} \right) \\ 0 \text{ otherwise} \end{cases}$$

$$h(0) = h_d(0) \times W_{Hann}(0) = \frac{3}{4} \times 1 = 0.75$$

$$h(1) = h_d(1) \times W_{Hann}(1) = -0.225 \times (0.905) = -0.204$$

$$h(2) = h_d(2) \times W_{Hann}(2) = (-0.159) \times 0.655 = -0.104$$

$$h(3) = h_d(3) \times W_{Hann}(3) = (-0.075) \times 0.345 = -0.026$$

$$h(4) = h_d(4) \times W_{Hann}(4) = 0 \times 0.8145 = 0$$

$$h(5) = h_d(5) \times W_{Hann}(5) = 0.045 \times 0 = 0$$

$$h(1) = h(-1); \quad h(2) = h(-2); \quad h(3) = h(-3);$$

$$h(4) = h(-4); \quad h(5) = h(-5)$$

$$H(z) = 0.75 - 0.204 [z^{-1} + z] - 0.104 (z^{-2} + z^2) - 0.026 (z^{-3} + z^3)$$

$$H(e^{j\omega}) = 0.75 - 0.408 \cos \omega - 0.208 \cos 2\omega - 0.052 \cos 3\omega$$

3
5

$$H\left(\frac{2\pi k}{15}\right) = 1; \text{ for } k=0, 1, 2, 3$$

$$= 0 \text{ for } k=4, 5, 6, 7$$

Modified discrete frequency response is

$$H\left(\frac{2\pi k}{15}\right) = \begin{cases} 1 \cdot e^{-j7\left(\frac{2\pi k}{15}\right)} & ; k=0, 1, 2, 3 \\ 0 & ; k=4, 5, 6, 7, 8, 9, 10 \\ 1 \cdot e^{j7\left(\frac{2\pi k}{15}\right)} & ; k=12, 13, 14. \end{cases}$$

$$h(n) = \frac{1}{N} \left[H(0) + 2 \operatorname{Re} \sum_{k=1}^{\frac{N-1}{2}} \left[H(k) \cdot e^{j\frac{2\pi nk}{N}} \right] \right]$$

$$= \frac{1}{15} \left[H(0) + 2 \operatorname{Re} \sum_{k=1}^3 \left(e^{-j\frac{14\pi k}{15}} \cdot e^{j\frac{2\pi nk}{15}} \right) \right]$$

$$= \frac{1}{15} \left[H(0) + 2 \operatorname{Re} \sum_{k=1}^3 e^{j\frac{2\pi k}{15}(n-7)} \right]$$

$$= \frac{1}{15} \left[1 + 2 \operatorname{Re} \left[e^{j\frac{2\pi}{15}(n-7)} \right] + 2 \operatorname{Re} \left[e^{j\frac{4\pi}{15}(n-7)} \right] + 2 \operatorname{Re} \left[e^{j\frac{6\pi}{15}(n-7)} \right] \right]$$

$$h(n) = \frac{1}{15} \left[1 + 2 \cos \frac{2\pi}{15}(n-7) + 2 \cos \frac{4\pi}{15}(n-7) + 2 \cos \frac{6\pi}{15}(n-7) \right]$$

$$h(0) = -0.05 = h(14) \quad h(5) = 0.0339 = h(9)$$

$$h(4) = 0.0411 = h(13) \quad h(6) = 0.3150 = h(8)$$

$$h(2) = 0.0668 = h(12) \quad h(7) = 0.4669$$

$$h(3) = -0.0364 = h(11) \quad h(7) = 0.4669$$

$$h(4) = -0.1079 = h(10)$$

$$H(z) = \sum_{n=0}^{14} h(n) \cdot z^{-n} \quad (11)$$

$$= -0.05(1 + z^{-14}) + 0.0411(z^{-1} + z^{-13}) +$$

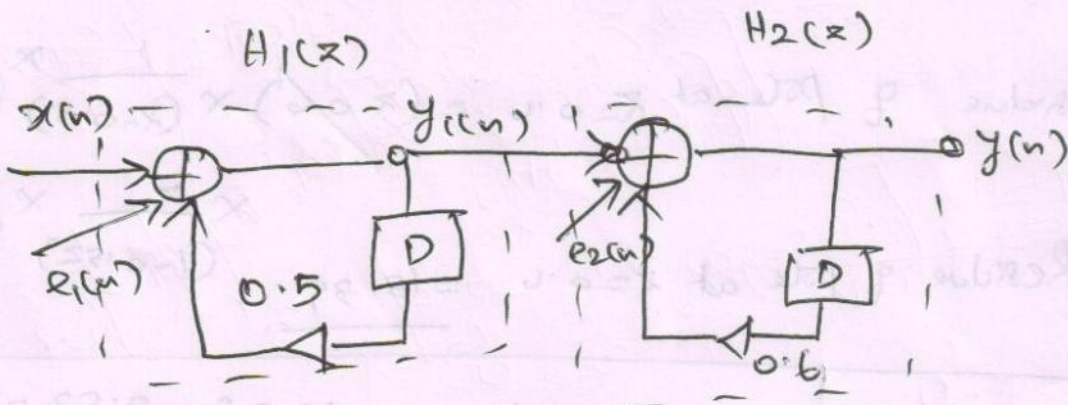
$$0.0668(z^{-2} + z^{-12}) + 0.0364(z^{-3} + z^{-11})$$

$$+ 0.1079(z^{-4} + z^{-10}) + 0.0339(z^{-5} + z^{-9})$$

$$+ 0.9190(z^{-6} + z^{-8}) + 0.4669 z^{-7}$$

(14)
a)

$$H_1(z) = \frac{1}{(1 - 0.5z^{-1})} \quad H_2(z) = \frac{1}{(1 - 0.6z^{-1})}$$



$$\sigma_{err_T}^2 = \sum_{k=1}^2 \sigma_{ok}^2 = \sigma_{e01}^2 + \sigma_{e02}^2$$

Evaluation of σ_{e01}^2

The first error source should see the entire transfer function.

$$H(z) = H_1(z) \cdot H_2(z) = \frac{z}{z - 0.5} \times \frac{z}{z - 0.6}$$

$$\sigma_{e02}^2 = \sigma_e^2 \times \frac{1}{2\pi j} \oint_C H(z) \cdot H(z^*) z^{-1} dz \quad (z - 0.5) \quad (z - 0.6)$$

$$\sigma_{e01}^2 = \sigma_e^2 \times \frac{1}{2\pi j} \oint_C \left[\frac{z}{(z - 0.5)} \times \frac{z}{(z - 0.6)} \right] \left[\frac{z^*}{(z^* - 0.5)} \times \frac{z^*}{(z^* - 0.6)} \right]$$

$$= \sigma_e^2 \times \left(\frac{1}{2\pi j} \oint_C \frac{z}{(z - 0.5)} \cdot \frac{1}{(z - 0.6)} \times \frac{1}{(1 - 0.5z^*)} \times \frac{z^*}{(1 - 0.6z^*)} dz \right)$$

Evaluation of the integral

There are four poles $z = 0.5, 0.6, 2, 1.67$

Two poles are less than one (0.5) & Two poles ($2, 1.67$) are greater than one.

Residue of pole at $z=0.5$

$$= (z-0.5) \times \frac{1}{(z-0.5)} \times \frac{1}{(z-0.6)} \times \frac{z}{(1-0.5z)}$$

$$\text{Residue of pole at } z=0.5 = -9.52$$

$$\text{Residue of pole at } z=0.6 = (z-0.6) \times \frac{1}{(z-0.5)} \times \frac{1}{(z-0.6)}$$

$$\text{Residue of pole at } z=0.6 = 13.39$$

$$\text{Evaluation of Integral is } = 13.39 - 9.52 = 3.87$$

$$\sigma_{e_1}^2 = \sigma_e^2 \times 3.87$$

Evaluation of $\sigma_{e_2}^2$

$$\sigma_{e_2}^2 = \sigma_e^2 \times \oint_C H(z) \cdot H(z^{-1}) \cdot z^{-1} dz$$

$H(z) = H_2(z)$ since second error source

uses only the second transfer function.

$$H(z) = \frac{z}{(z-0.6)} ; H(z^{-1}) = \frac{z^{-1}}{(z^{-1}-0.6)}$$

$$\sigma_{e_2}^2 = \sigma_e^2 \times \frac{1}{2\pi j} \oint \frac{z}{(z-0.6)} \times \frac{z^{-1}}{(z^{-1}-0.6)} \times z^{-1} dz$$

Evaluation of Integral is = Sum of Residues of poles less than unity ⁽¹³⁾

There are two poles $z=0.6, 1.67$ among which only one pole $z=0.6$ is less than unity.

Residue of pole at $z=0.6 = (z/0.6) \times \frac{1}{(z/0.6) - 0.6} \times \frac{1}{z=0.6}$

Residue of pole at $z=0.6 = \frac{0.6}{0.64} = 0.9375$

$$\sigma_{e02}^2 = \sigma_{e2}^2 \times 0.9375$$

$$\sigma_{err}^2 = \sigma_{e01}^2 + \sigma_{e02}^2 = 3.87 \sigma_{e2}^2 + 0.9375 \sigma_{e2}^2$$

$$\sigma_{err}^2 = 4.8075 \sigma_{e2}^2$$

(e)h)

$$y(n) = 0.95y(n-1) + x(n);$$

Note: In this problem initial condition not given. We have to assume it.

$x(n) = 0$; Assume $y(-1) = 13$

(Note: In this problem initial condition is not given. we assume it)

n	un-quantized $y(n)$	Rounded $y(n)$	
0	12.35	12	Dead band $= \frac{1}{2}$ $\frac{1}{1- a }$ $= \frac{1}{2}$ $\frac{1}{1-(0.95)}$ $= \frac{0.9}{0.05}$ $= [-10, 10]$
1	11.4	11	
2	10.45	10	
3	9.5	10	
4	9.05	10	
5	9.5	10	

15) a)

Refer the class notes & Book.

15) b)

Refer the class note & Book.

$$P_{n+1} = \frac{P_n + P_{n-1}}{2}$$

$$P_{n+1} = \frac{P_n + P_{n-1}}{2}$$

$$P_{n+1} = \frac{P_n + P_{n-1}}{2}$$

$$P_{n+1} = \frac{P_n + P_{n-1}}{2}$$

Note: In this program...

$$f(n) = 0.5f(n-1) + a(n)$$

$$f(-1) = 13$$

(Note: In this problem graph...

n	UN (n)	UN (n-1)	UN (n-2)
0	12.52	12	12
1	11.4	11	11
2	10.42	10	10
3	9.2	10	10
4	8.2	10	10
5	7.2	10	10