Question Paper Code : 40962

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2018

Fifth/Sixth Semester

Electronics and Communication Engineering

EC 6502 – PRINCIPLES OF DIGITAL SIGNAL PROCESSING

(Common to Biomedical Engineering/Medical Electronics)

(Regulations 2013)

Time : Three Hours | Maximum : 100 Marks

Answer ALL questions

PART – A (10×2=20 Marks)

1. Calculate the 4-point DFT of the sequence $x(n) = \left\{\underset{\uparrow}{1} \quad 0 \quad -1 \quad 0\right\}$.

2. What is the relationship between Fourier transform and DFT ?

3. What are the methods used for digitizing the analog filter into a digital filter ?

4. What is meant by frequency warping ?

5. Draw the direct form realization of FIR system.

6. How the zeros in FIR filter is located ?

7. Distinguish between fixed point arithmetic and floating point arithmetic.

8. Why is rounding preferred over truncation in realizing a digital filter ?

9. Show that the up sampler and down sampler are time invariant system.

10. Write the expression for the output y(n) as a function of the input x(n) for the given multirate system as in Figure 1.

$$x(n) \rightarrow \boxed{\uparrow 5} \rightarrow \boxed{\downarrow 10} \rightarrow \boxed{\uparrow 2} \rightarrow y(n)$$

Figure 1

(2)

PART – B (5×13=65 Marks)

11. a) i) State and prove any four properties of DFT. (8)

ii) Perform circular convolution of the following sequences $x_1(n) = \{1 \ 1 \ 2 \ 1\}$; $x_2(n) = \{1 \ 2 \ 3 \ 4\}$. (5)

(OR)

b) i) Mention the differences and similarities between DIT and DIF algorithms. (5)

ii) Compute 4 point DFT of a sequence $x(n) = \{0 \ 1 \ 2 \ 3\}$ using DIF and DIT algorithms. (8)

12. a) i) Design an analog Butterworth filter for a given specifications. (7)

$0.9 \le |H(j\Omega)| \le 1$ for $0 \le \Omega \le 0.2\pi$.

$|H(j\Omega)| \le 0.2$ for $0.4\pi \le \Omega \le \pi$.

ii) Apply impulse invariant method and find H(z) for $H(s) = \dfrac{s+a}{(s+a)^2 + b^2}$. (6)

(OR)

b) i) Apply bilinear transformation to $H(s) = \dfrac{2}{(s+1)(s+2)}$ with T = 1 sec and find H(z). (6)

ii) Explain the Lattice-Ladder structure with neat diagram. (7)

13. a) Write the expression for the frequency response of Rectangular window and Hamming window and explain. (7+6)

(OR)

b) Determine the filter coefficients h(n) obtained by sampling

$$H_d(e^{j\omega}) = e^{-j(N-1)\omega/2} \quad 0 \le |\omega| \le \frac{\pi}{2}$$

$$= 0 \quad \frac{\pi}{2} \le |\omega| \le \pi$$

for N = 7. (13)

14. a) The output signal of an A/D convertor is passed through a first order low pass filter, with transfer function given by $H(z) = \dfrac{(1-a)z}{z-a}$ for $0 \le a \le 1$. Find the steady state output noise power due to quantization at the output of the digital filter. (13)

(OR)

b) Briefly explain the following :

i) Coefficient quantization error. (4)

ii) Product quantization error. (4)

iii) Truncation and Rounding. (5)

③

15. a) Explain sampling rate conversion by a rational factor and derive input-output relation in both time and frequency domain. (13)

(OR)

b) With neat required diagrams explain any two applications of adaptive filtering. (6+7)

PART – C (1×15=15 Marks)

16. a) An FIR Filter is given by the difference equation

$$y(n) = 2x(n) + \frac{4}{5}x(n-1) + \frac{3}{2}x(n-2) + \frac{2}{3}x(n-3)$$

Determine its lattice form. (15)

(OR)

b) How is signal scaling used to prevent overflow limit cycle in the digital filter implementation ? Explain with an example. (15)

April – May 2018

1. **Calculate the 4 – point DFT of the sequence x(n) = { 1,0,-1,0}**

   $X(K) = \{ 0, 2, 0, 2\}$

2. **What is relationship between Fourier Transform and DFT?**

$$X(e^{j\omega}) \longrightarrow X\left(\frac{2\pi k}{N}\right) \longrightarrow X(K)$$

$$\omega_K = \frac{2\pi K}{N}$$

3. **What are the methods used for digitizing the analog filter into digital filter?**

   1. Bilinear Transformation
   2. Impulse Invariant Transformation

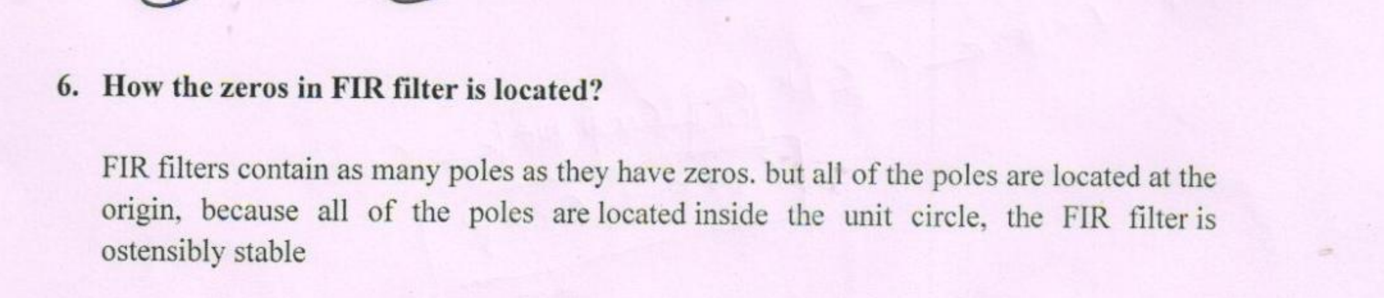
4. **What is frequency Warping ?**

   In bilinear transformation the relation between analog and digital frequencies is ($w = 2 \tan^{-1}\Omega T$) non- linear. When S plane is mapped into Z plane using bilinear transformation, the non- linear relationship introduces distortion in frequency axis which is called frequency warping.

5. **Draw the direct form realization of FIR system**

$$H(z) = \sum_{n=0}^{N-1} h(n) \cdot z^{-n}$$

$$y(n) = h(0)\cdot x(n) + h(1)\, x(n-1) + h(2)\, x(n-2) + \cdots + h(N-1)\cdot x(n-N-1)$$



6. **How the zeros in FIR filter is located?**

   FIR filters contain as many poles as they have zeros. but all of the poles are located at the origin, because all of the poles are located inside the unit circle, the FIR filter is ostensibly stable

7. Distinguish between fixed point arithmetic and floating point arithmetic?

Fixed Point Arithmetic	**Floating Point Arithmetic**
Accuracy of the result is less due to smaller dynamic range	Accuracy of the result is high due to larger dynamic range
Speed of processing is high	Speed of processing is low
Hardware implementation is cheaper	Hardware implementation is economical
It can be used for real time applications	It cannot be used for real time applications

8. Why rounding preferred over truncation in realizing digital filter ?

1. The rounding error is independent of the type of arithmetic
2. The mean value of the rounding error signal is zero
3. The variance of the rounding error signal is least

9. Show that the up sampler and down sampler are time invariant (Variant) systems.

Down sampler:

$y(n) = x(\tfrac{1}{2}n)$

Apply shifted input $x(n-n_0)$

$y(n, n_0) = x(\tfrac{1}{2}n - n_0)$ — ①

Shift the entire equation by n_0

$y(n-n_0) = x(\tfrac{1}{2}n - \tfrac{1}{2}n_0)$ — ②

$1 \neq 2$ are not equal

Time variant system.

Up sampler:

$y(n) = x(2n)$

Apply shifted input

$y(n, n_0) = x(2n - n_0)$ — ①

Shift the entire output

$y(n-n_0) = x(2n - 2n_0)$ — ②

1 and ② are not equal.

Time variant system.

10. Write the expression for the output y(n) as a function of the input x(n) for the given multi rate system as in figure 1

$$x(n) \rightarrow \boxed{\uparrow 5} \rightarrow \boxed{\downarrow 10} \rightarrow \boxed{\uparrow 2} \rightarrow y(n)$$

$$x(5n) \rightarrow x\left(\tfrac{n}{2}\right) \rightarrow x(n)$$

$$\boxed{y(n) = x(n).}$$

PART B

11) a) i) State and Prove any four Properties of DFT

(Refer Book (or) class notes)

11) a) ii) Circular Convolution of the sequences.

$$x_1(n) = \{1, 1, 2, 1\}$$

$$x_2(n) = \{1, 2, 3, 4\}$$

Matrix Method:

$x_1(n)$ is unshifted & $x_2(n)$ is shifted

$$x_3(n) = \begin{bmatrix} x_2(0) & x_2(3) & x_2(2) & x_2(1) \\ x_2(1) & x_2(0) & x_2(3) & x_2(2) \\ x_2(2) & x_2(1) & x_2(0) & x_2(3) \\ x_2(3) & x_2(2) & x_2(1) & x_2(0) \end{bmatrix} \begin{bmatrix} x_1(0) \\ x_1(1) \\ x_1(2) \\ x_1(3) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 13 \\ 14 \\ 11 \\ 12 \end{bmatrix}$$

$$x_3(n) = \{\underset{\uparrow}{13}, 14, 11, 12\}$$

11) b) Similarities between DIF & DIT:

i) The value of N should be expressed such that $N = 2^m$ and both algorithm consists of 'm' stages of computations.

ii) The Number of butterfly diagrams are additions are $N \log_2^N$ and total number of complex multiplications are $\frac{N}{2} \log_2^N$.

Difference between DIT & DIF

DIT - FFT	DIF - FFT
Time domain sequence is decimated.	Frequency domain sequence is decimated
Input should be in bit reversed order and output will be in normal order	The Input should be in normal order and output will be in reversed bit order.
In each stage of computation, the phase factors are multiplied before add and subtract operations.	In each stage of computation the phase factors are multiplied after add and subtract operations.

12) a) Design butterworth filter:

$$0.9 \le |H(j\Omega)| \le 1 ; 0 \le \Omega \le 0.2\pi$$

$$|H(j\Omega)| \le 0.2 ; 0.4\pi \le \Omega \le \pi$$

Bilinear Transformation; $T=1$

Soln:

$\delta_1 = 0.9$; $\delta_2 = 0.2$; $\omega_1 = 0.2\pi$; $\omega_2 = 0.4\pi$

$$\Omega_1 = \frac{2}{T}\tan\frac{\omega_1}{2} = 2\tan\frac{0.2\pi}{2} = 0.649$$

$$\Omega_2 = \frac{2}{T}\tan\frac{\omega_2}{2} = 2\tan\frac{0.4\pi}{2} = 1.453$$

$$\boxed{\frac{\Omega_2}{\Omega_1} = \frac{1.453}{0.649} = 2.238}$$

order of the filter

$$N \geq \frac{1}{2}\frac{\log\left[\left\{\left(\frac{1}{\delta_2^2}\right)-1\right\}\Big/\left\{\left(\frac{1}{\delta_1^2}\right)-1\right\}\right]}{\log\left(\frac{\Omega_2}{\Omega_1}\right)}$$

$$N \geq \frac{1}{2}\frac{\log\left[24/0.234\right]}{\log\left[2.238\right]} = \frac{1}{2}\times\frac{2.009}{0.349}$$

$$\boxed{N \geq 2.86 = 3}$$

$$\Omega_c = \frac{\Omega_1}{\left[\left(\frac{1}{\delta_1^2}\right)-1\right]^{\frac{1}{2N}}} = \frac{0.649}{\left[0.234\right]^{\frac{1}{6}}} = \frac{0.649}{0.7849}$$

$$\boxed{\Omega_c = 0.826 \text{ rad/s}}$$

N = odd; 3; The transfer function is

$$H_a(s) = \left[\frac{B_0\Omega_c}{s+C_0\Omega_c}\right]\prod_{k=1}^{(N-1)/2}\frac{B_k\Omega_c^2}{s^2+b_k\Omega_c s+C_k\Omega_c^2}$$

$$= \left[\frac{B_0\Omega_c}{s+C_0\Omega_c}\right]\left[\frac{B_1\Omega_c^2}{s^2+b_1\Omega_c s+C_1\Omega_c^2}\right]$$

$B_0 = B_1 = C_0 = C_1 = 1$

$$H_a(s) = \left[\frac{0.826}{s+0.826}\right]\left[\frac{1\times(0.826)^2}{s^2+1\times 0.826s+(0.826)^2}\right]$$

$$= \left[\frac{0.826}{s+0.826}\right]\left[\frac{0.682}{s^2+0.826s+0.682}\right]$$

Apply bilinear Transformation.

$$s = \frac{2}{T}\left(\frac{z-1}{(z+1)}\right) = 2\left[\frac{z-1}{z+1}\right]$$

$$= \left[\frac{0.826}{2\left[\frac{z-1}{z+1}\right]+0.826}\right]\left[\frac{0.682}{4\left[\frac{z-1}{z+1}\right]^2+0.826\times 2\left(\frac{z-1}{z+1}\right)+0.682}\right]$$

$$= \left[\frac{0.826(z+1)}{2(z-1)+0.826(z+1)}\right]\left[\frac{0.682(z+1)^2}{4(z-1)^2+1.652(z^2-1)+0.682(z+1)^2}\right]$$

12) a) ii)

$$H(s) = \frac{(s+a)}{(s+a)^2+b^2}$$

Apply impulse invariant transformation

$$H(z) = \frac{1-e^{-aT}\cos bT z^{-1}}{1-2e^{-aT}\cos bT z^{-1}+e^{-2aT}\cdot z^{-2}}$$

12) b) i)

$$H(s) = \frac{2}{(s+1)(s+2)} \quad T=1 \quad ; \text{ Method : bilinear Transformation}$$

Soln:

$$s = \frac{2}{T}\left(\frac{z-1}{z+1}\right) = 2\left(\frac{z-1}{(z+1)}\right)$$

$$H(z) = \frac{2}{\left[2\left(\frac{z-1}{z+1}\right)+1\right]\left[\frac{2(z-1)}{(z+1)}+2\right]}.$$

$$= \frac{2}{\frac{\left[2(z-1)+(z+1)\right]}{(z+1)}\left[\frac{2(z-1)+2(z+1)}{(z+1)}\right]}$$

$$= \frac{2(z+1)^2}{\left[2(z-1)+(z+1)\right]\left[2(z-1)+2(z+1)\right]}$$

$$H(z) = \frac{2(z+1)^2}{(3z-1)(4z-1)}$$

---

12) b) ii) Lattice – Ladder structure with neat diagram

(Refer the book: Page 561)

(DSP by Salivahanan)

---

13) a) Expression for frequency Response of Rectangular & Hamming window:.

Frequency Response equation for Rectangular window:.

$$W_R(e^{j\omega}) = e^{-j\omega\frac{(M-1)}{2}}\left[\frac{\sin\left(\frac{\omega M T}{2}\right)}{\sin\left(\frac{\omega T}{2}\right)}\right]$$

Hamming window:.

$$W_H(e^{j\omega}) = 0.54\,\frac{\sin\left(\frac{MT}{2}\right)}{\sin\left(\frac{\omega T}{2}\right)} - 0.46\,\frac{\sin\left(\frac{\omega M T}{2} - \frac{M\pi}{(M-1)}\right)}{\sin\left(\frac{\omega T}{2} - \frac{\pi}{(M-1)}\right)}$$

further explanation will be obtained from the book DSP by Salivahanan: (Page 449 - 451)

13) b)

$$H_d(e^{j\omega}) = e^{-j\left(\frac{N-1}{2}\right)\omega} \quad 0 \le |\omega| \le \pi/2$$

$$= 0 \quad \frac{\pi}{2} \le |\omega| \le \pi$$

for $N=7$; frequency sampling Technique:

frequency Response can be written as

$$H_d(e^{j\omega}) = e^{-j3\omega} \quad 0 \le \omega \le \pi/2$$

$$= e^{-j3\omega} \quad -\frac{\pi}{2} \le \omega \le 0$$

$$= 0 \quad \frac{\pi}{2} \le |\omega| \le \pi$$

and it can further re written as

$$H_d(e^{j\omega}) = e^{-j3\omega} \quad -\frac{\pi}{2} \le \omega \le \pi/2$$

$$= 0 \quad \frac{\pi}{2} \le |\omega| \le \pi$$

It is enough to take only one half of the frequency response.

$$H_d(e^{j\omega}) = e^{-j3\omega} \quad 0 \le \omega \le \frac{\pi}{2}$$

$$= 0 \quad \frac{\pi}{2} \le |\omega| \le \pi$$

Take $\omega_K = \frac{2\pi K}{N} = \frac{2\pi K}{7}$; $K = 0, 1, 2, 3 \ldots 6$

$K=0 \Rightarrow \omega_K = \frac{2\pi}{7}\times 0 = 0$

$K=1 \Rightarrow \omega_K = \frac{2\pi}{7} = 0.897$

$K=2 \Rightarrow \omega_K = \frac{4\pi}{7} = 1.795$

$K=3 \Rightarrow \omega_K = \frac{6\pi}{7}$

$K=4 \Rightarrow \omega_K = \frac{8\pi}{7}$

$K=5 \Rightarrow \omega_K = \frac{10\pi}{7}$

$K=6 \Rightarrow \omega_K = \frac{12\pi}{7}$

For the values of $K=0,1,6$; the desired frequency response is non zero & for other values frequency response is zero. $K=1$; & $K=6$ are complex conjugates of each other.

$$H\left(\frac{2\pi K}{7}\right) = \begin{cases} e^{-j\frac{6\pi K}{7}} & K=0,1 \\ 0 & K=2,3,4,5 \\ e^{j\frac{6\pi K}{7}} & K=6 \end{cases}$$

$$h(n) = \frac{1}{N}\left\{H(0) + \sum_{K=1}^{\left(\frac{N-1}{2}\right)} 2\,\mathrm{Re}\left\{H(K)\cdot e^{j\frac{2\pi nK}{N}}\right\}\right\}$$

$$= \frac{1}{7}\left\{1 + \sum_{K=1} 2\,\mathrm{Re}\left\{e^{-j\frac{6\pi K}{7}}\cdot e^{j\frac{2\pi nK}{7}}\right\}\right\}$$

$$= \frac{1}{7}\left\{1 + \sum_{K=1} 2\,\mathrm{Re}\left\{e^{j\frac{2\pi}{7}K(n-3)}\right\}\right\}$$

$$h(n) = \frac{1}{7}\left\{1 + 2\cos\frac{2\pi}{7}(n-3)\right\}$$

$$h(0) = \frac{1}{7}\left\{1 + 2\cos\frac{6\pi}{7}\right\} = -0.1146 = h(6)$$

$= 0.0793 = h(5)$

$$h(2) = \frac{1}{7}\left\{1 + 2\cos\frac{2\pi}{7}\right\} = 0.321 = h(4)$$

$$h(3) = \frac{1}{7}\left\{1+2\right\} = 0.4286$$

$$H(z) = \sum_{n=0}^{6} h(n)\cdot z^{-n}$$

$$= -0.1146\left[1+z^{-6}\right] + 0.0793\left[z^{-1}+z^{-5}\right] +$$

$$0.321\left[z^{-2}+z^{-4}\right] + 0.4286\left[z^{-3}\right]$$

---

(4) a) $$H(z) = \frac{z(1-a)}{(z-a)} \quad 0 \le a \le 1$$

Soln:
$$\sigma_{eo}^2 = \sigma_e^2 \times \frac{1}{2\pi j}\oint_c H(z)\cdot H(z^{-1})\cdot z^{-1}\, dz$$

$$H(z) = \frac{z(1-a)}{(z-a)}; \quad H(z^{-1}) = \frac{z^{-1}(1-a)}{(z^{-1}-a)}$$

$$\sigma_{eo}^2 = \sigma_e^2 \times \frac{1}{2\pi j}\oint_c \frac{z(1-a)}{(z-a)} \times \frac{z^{-1}(1-a)}{(z^{-1}-a)} \times z^{-1}\, dz$$

Multiply Second Transfer by 'z' in numerator & denominator

$$= \sigma_e^2 \times \boxed{\frac{1}{2\pi j}\oint_c \frac{(1-a)}{(z-a)} \times \frac{(1-a)}{(1-az)}\, dz}$$

This integral can be evaluated by Cauchy's method. The evaluation is equal to sum of Residues. The evaluation is less than unity. Residues of poles which where as there are two poles $z=a$; & $z=\frac{1}{a}$ is greater than 'a' is less than one & $\frac{1}{a}$ is greater than one. The evaluation of the integral is equal one. The evaluation ... at $z=a$.

$$\left.\begin{array}{l}\text{Residue at } z=a\end{array}\right\} = \cancel{(z-a)} \times \frac{(1-a)}{\cancel{(z-a)}} \times \frac{(1-a)}{(1-az)}\Bigg|_{z=a}$$

$$= \frac{(1-a)^2}{(1-a^2)} = \frac{(1-a)^2}{\cancel{(1-a)}(1+a)}$$

$$\text{Residue of the pole} = \frac{(1-a)}{(1+a)}$$

$$\boxed{\sigma_{e_0}^2 = \sigma_e^2 \times \frac{(1-a)}{(1+a)}} \quad \text{where } \sigma_e^2 = \frac{2^{-2b}}{12}$$

$b \rightarrow$ number of bits after quantization.

14) b) Briefly explain the following

i) Co-efficient quantization error

ii) Product quantization error

iii) Truncation & Rounding

(Please refer the book & class notes).

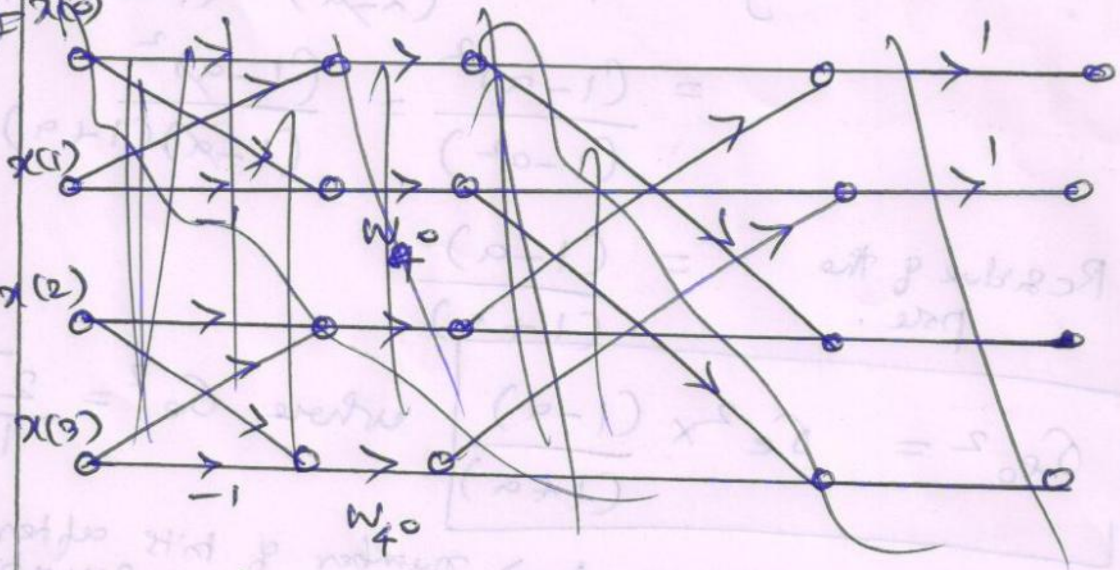
15) a) 15 b) for both questions Please Refer the book & class notes.

16) a) out of syllabus question.

16) b) Refer the book (DSP by Salivahanan) (Page No: 596-597.)

→ End ←

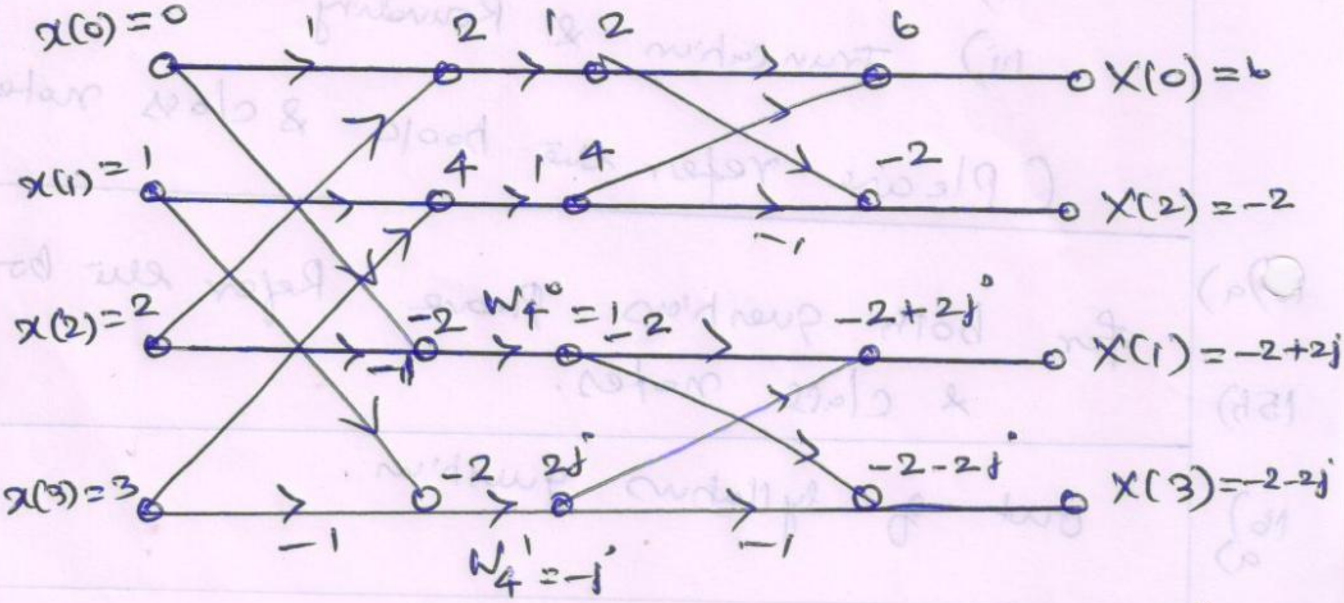
~~12) b) ii)~~



12)
b) ii)

$$x(n) = \{0, 1, 2, 3\}$$

$$W_4^0 = 1 \; ; \quad W_4^1 = -j$$



$$X(K) = \{6, -2+2j, -2, -2-2j\}$$