Frequency Domain Sampling DFT

1. If x(n) is a finite duration sequence of length L, then the discrete Fourier transform X(k) of x(n) is given as:

$$a)\ \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi kn}{N}} (L < N)(k = 0,1,2,3 \ldots N-1)$$

$$b)\ \sum_{n=0}^{N-1} x(n) e^{\frac{j2\pi kn}{N}} (L < N)(k = 0,1,2,3 \ldots N-1)$$

$$c)\ \sum_{n=0}^{N-1} x(n) e^{\frac{j2\pi kn}{N}} (L > N)(k = 0,1,2,3 \ldots N-1)$$

$$d)\ \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi kn}{N}} (L > N)(k = 0,1,2,3 \ldots N-1)$$

Answer: a

Explanation: If x(n) is a finite duration sequence of length L, then the Fourier transform of x(n) is given as

$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

If we sample X(ω) at equally spaced frequencies ω=2πk/N, k=0,1,2…N-1 where N>L, the resultant samples are

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi kn}{N}}$$

2. If X(k) discrete Fourier transform of x(n), then the inverse discrete Fourier transform of X(k) is:

$$a)\ \frac{1}{N} \sum_{k=0}^{N-1} X(K) e^{\frac{-j2\pi kn}{N}}$$

$$b)\ \sum_{k=0}^{N-1} X(K) e^{\frac{-j2\pi kn}{N}}$$

$$c)\ \sum_{k=0}^{N-1} X(K) e^{\frac{j2\pi kn}{N}}$$

$$d)\ \frac{1}{N} \sum_{k=0}^{N-1} X(K) e^{\frac{j2\pi kn}{N}}$$

Answer: d

Explanation: If X(k) discrete Fourier transform of x(n), then the inverse discrete Fourier transform of X(k) is given as

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(K) e^{\frac{j2\pi kn}{N}}$$

Frequency Domain Sampling DFT

3. The Nth rot of unity W_N is given as:

a) $e^{j2\pi N}$ b) $e^{-j2\pi N}$ c) $e^{-j2\pi/N}$ d) $e^{j2\pi/N}$

Answer: c

Explanation: We know that the Discrete Fourier transform of a signal x(n) is given as

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi kn}{N}} = \sum_{n=0}^{N-1} x(n) W_N^{Kn}$$

Thus we get Nth rot of unity $\mathbf{W_N= e^{-j2\pi/N}}$

4. Which of the following is true regarding the number of computations requires to compute an N-point DFT?

a) N^2 complex multiplications and N(N-1) complex additions
b) N^2 complex additions and N(N-1) complex multiplications
c) N^2 complex multiplications and N(N+1) complex additions
d) N^2 complex additions and N(N+1) complex multiplications

Answer: a

Explanation: The formula for calculating N point DFT is given as

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi kn}{N}}$$

From the formula given at every step of computing we are performing N complex multiplications and N-1 complex additions. So, in a total to perform N-point DFT we perform N^2 complex multiplications and N(N-1) complex additions.

5. Which of the following is true?

$\boldsymbol{a}$) $W_{N^*} = \frac{1}{N} W_N^{-1}$ $\boldsymbol{b}$) $W_N^{-1} = \frac{1}{N} W_N^{*}$

$\boldsymbol{c}$) $W_N^{-1} = W_{N^*}$ $\boldsymbol{d}$) None of the mentioned

Answer: b

Explanation: If XN represents the N point DFT of the sequence xN in the matrix form, then we know that

Frequency Domain Sampling DFT

$$X_N = W_N x_N$$

By pre-multiplying both side by W_N , We get

$\mathbf{x_N} = \boldsymbol{W_N^{-1}}\mathbf{X_N}$

But we know that the inverse DFT of X_N is defined as,

$\mathbf{x_N} = \frac{1}{N}\boldsymbol{W_N^{*}}\mathbf{X_N}$

Thus by comparing above two equations, we get

$$\boldsymbol{W_N^{-1} = \frac{1}{N} W_N^{*}}$$

6. What is the DFT of the four point sequence x(n)={0,1,2,3}?
   a) {6,-2+2j-2,-2-2j}
   b) {6,-2-2j, 2,-2+2j}
   c) {6,-2+2j,-2,-2-2j}
   d) {6,-2-2j,-2,-2+2j}

   Answer: c
   Explanation: The first step is to determine the matrix W4. By exploiting the periodicity property of W4 and the symmetry property

   $$\mathbf{W_N^{k+N/2} = -W_N^{k}}$$

   The matrix W_4 may be expressed as

$$W_4=\begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^0 & W_4^2 \\ W_4^0 & W_4^3 & W_4^2 & W_4^1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$\text{Then } X_4=W_4.x_4=\begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

7. If X(k) is the N point DFT of a sequence whose Fourier series coefficients is given by c_k, then which of the following is true?

   a) $X(k)=Nc_k$
   b) $X(k)=c_k/N$
   c) $X(k)=N/c_k$
   d) None of the these

   Answer: a
   Explanation: The Fourier series coefficients are given by the expression

Frequency Domain Sampling DFT

$$c_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi kn}{N}}$$

$$c_k = \frac{1}{N}X(K)$$

$$\boldsymbol{X(K) = Nc_k}$$

8. What is the DFT of the four point sequence x(n)={0,1,2,3}?

a) {6,-2+2j-2,-2-2j}
b) {6,-2-2j,2,-2+2j}
c) {6,-2-2j,-2,-2+2j}
d) {6,-2+2j,-2,-2-2j}

Answer: d

Answer: Given x (n)={0,1,2,3}

We know that the 4-point DFT of the above given sequence is given by the expression

$$\boldsymbol{X(K) = \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi kn}{N}}}$$

In this case N=4

=>**X(0)=6,X(1)=-2+2j,X(2)=-2,X(3)=-2-2j.**

9. If $W_4^{100}=W_x^{200}$, then what is the value of x?

a) 2
b) 4
c) 8
d) 16

Answer: c

Explanation: We know that according to the periodicity and symmetry property,

$$\frac{100}{4} = \frac{200}{x}$$

$$x = \frac{200}{100} * 4$$

$$\boldsymbol{x = 8}$$